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Spurious rejection of the stationarity hypothesis in the presence of a break point

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It is shown that KPSS and LMC tests may be seriously biased when there is a shift in the level or in the trend of the time series under study.

I. INTRODUCTION

Structural change in time series

In the last few years, many studies have been developed on testing the null of a unit root when there is a structural change in the time series. As Perron (1989) demonstrated, the Dickey–Fuller (1979) test (DF) is not very powerful when the true data generating process (d.g.p.) is stationary around a broken linear trend. To avoid this drawback, Perron and others develop tests based on the introduction of dummy variables in the testing equation. Campos *et al.* (1996) show that, if the DF test is applied, a first order integrated process, $I(1)$, could seem as an $I(2)$ process when the true d.g.p. is an $I(1)$ with a break, which is the so called *Perron phenomenon*. However, Leybourne *et al.* (1998) show a different result, i.e. when the DF test is routinely applied to processes with structural breaks, an $I(1)$ process could seem to be a stationary one. Moreover, when the break point can be treated as exogenous, Perron (1989, 1993, 1994) unit root tests are more adequate, since those tests allow for a break under the null and alternative hypotheses.

The goal of this paper is to analyse the effects on the size of KPSS[1] and LMC[2] tests, when the d.g.p. is stationary (around a level, or around a trend) with a structural change. Unlike Christiano (1988) and Banerjee *et al.* (1992) that consider an unknown break point, the present study works with the hypothesis of a known structural change point, as in Perron (1989) and Leybourne *et al.* (1998).

In Section II will be shown the main stationarity tests, i.e., the KPSS test and the LMC test. In Section III is shown the results from the simulation study intended to analyse the size and power properties of the KPSS and LMC tests when there is a structural change in the intercept or in the slope of the time series. In Section IV are presented the main conclusions from the study.

II. TESTS OF STATIONARITY IN TIME SERIES

KPSS test

To test for the null hypothesis of stationarity around a level or a trend through the KPSS test, the time series y_t is decomposed into the sum of a deterministic trend (t), a stochastic trend (r_t) and a stationary error term (ε_t):

$$y_t = \xi t + r_t + \varepsilon_t \tag{1}$$

where r_t is a random walk:

$$r_t = r_{t-1} + u_t \tag{2}$$

in which u_t is i.i.d $(0, \sigma_u^2)$. The initial value r_0 is treated as fixed and serves the role of an intercept. The stationarity hypothesis is $\sigma_u^2 = 0$. Under the H_0, if ε_t is assumed to be stationary and $\xi \neq 0$, y_t is trend stationary, whereas under the same assumptions, except for $\xi = 0$, y_t is stationary around a level (r_0).[3]

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[1] Kwiatkwoski *et al.* (1992).

[2] Leybourne *et al.* (1994).

[3] Nyblom and Makelainen (1983) give the locally best invariant statistic for the level-stationary case. Nyblom (1986) considers a model equivalent to the KPSS test giving this statistic.

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The statistic used is the LM statistic, which tests the null hypothesis $\sigma_u^2 = 0$, against the alternative of $\sigma_u^2 > 0$, under the assumptions that u_t is normal and that ε_t is i.i.d $N(0, \sigma_\varepsilon^2)$.

Let e_t be the residuals from the regression of y_t on an intercept[4] or a trend. Let $\hat{\sigma}_\varepsilon^2$ be the estimate of the error variance from this regression (the sum of squared residuals, divided by the sample size, T), the *partial sum of the residuals* is defined as:

$$S_t = \sum_{i=1}^{t} e_i \quad t = 1, 2, \ldots, T \tag{3}$$

The LM[5] statistic is:

$$LM = \sum_{t=1}^{T} S_t^2 / \hat{\sigma}_\varepsilon(l) \tag{4}$$

Under the stationarity hypothesis, this LM statistic converges to $\int_0^1 V_X^2$, where $V_X(r) = W(r) - [\int_0^r X'] \times [\int_0^1 XX']^{-1} [\int_0^1 X \,\mathrm{d}W]$ is a generalized Brownian bridge process. Note that the statistic LM depends on the sample size and the number of lags (l) of the error term (ε_t).

The LM statistic, which is shown in Equation 4, is derived under the assumption that the error term ε_t is i.i.d.$N(0, \sigma_\varepsilon^2)$. However, this assumption is not realistic in time series. Therefore, the appropriate denominator of the LM test statistic is a consistent estimator of the long term variance of ε_t, instead of $\hat{\sigma}_\varepsilon^2$. Thus, if the long term variance of ε_t is defined as:

$$\sigma^2 = \lim_{T \to \infty} T^{-1} E(S_T^2) \tag{5}$$

the consistent estimator of σ^2 is calculated from residuals e_t, called $s^2(l)$, as in Phillips (1987) or in Phillips and Perron (1988). Particularly, in this paper an estimator is used which has the same structure as in Kwiatkowski *et al.* (1992).

$$s^2(l) = T^{-1} \sum_{t=1}^{T} e_t^2 + 2T^{-1} \sum_{s=1}^{l} w(s, l) \sum_{t=s+1}^{T} e_t e_{t-s} \tag{6}$$

where $w(s, l)$ is an optional weighting function that corresponds to a spectral window.[6] Hence, the test statistic with the appropriate residual term is:[7]

$$\hat{\eta} = T^{-2} \sum S_t^2 / s^2(l) \tag{7}$$

in such a way that the H_0 is rejected if this statistic exceeds its critical value under the null.

LMC test

The LMC test also tests the H_0 of stationarity. This test is based on the same model as the KPSS test, although Leybourne and McCabe (1994) modify this model to include an autoregressive polynomial structure of the following form:

$$\Phi(L) y_t = \xi t + C + \varepsilon_t \tag{8}$$

in which $\Phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p$ is an autoregressive polynomial, L is the lag operator with roots outside the unit circle, p is the lags order,[8] ε_t is distributed as i.i.d. $(0, \sigma_\varepsilon^2)$.

Under some regularity conditions, the structural model (Equation 8) is a second-order equivalent to a reduced ARIMA(p,1,1) process (Harvey, 1989):

$$\Phi(L)(1 - L) y_t = \xi + (1 - \theta L) \zeta_t \quad 0 < \theta < 1 \tag{9}$$

in which ζ_t is distributed i.i.d.$(0, \sigma_\zeta^2)$, being $\sigma_\zeta^2 = \sigma_\varepsilon^2 / \theta$ and θ related to σ_u^2 in the following form:[9] $\theta = (\lambda + 2 - (\lambda^2 + 4\lambda)^{1/2})/2$, with $\lambda = \sigma_u^2 / \sigma_\varepsilon^2$. To test the H_0 of stationarity, ARIMA (p,0,0), against the alternative of a model ARIMA(p,1,1) with positive coefficients of the MA process, the null hypothesis is defined as $H_0 : \sigma_u^2 = 0$ against $H_1 : \sigma_u^2 > 0$.

To apply the LMC test, the series y_t^* is first constructed:

$$y_t^* = y_t - \sum_{s=1}^{p} \phi_s^* y_{t-s} \tag{10}$$

where the ϕ_s^* are the maximum likelihood estimates of ϕ_s, from the fitted ARIMA (p,1,1) model:

$$\Delta y_t = \xi + \sum_{s=1}^{p} \phi_i \Delta y_{t-i} + \zeta_t - \theta \zeta_{t-1}$$

Next, are calculated the residuals from the least squares regression of y_t^* against an intercept (if $\xi = 0$) and a time trend (if $\xi \neq 0$). If e_t is called the residuals of this regression, the LM statistic $(\hat{\nu})$ is the following:[10]

$$\hat{\nu} = T^{-2} \sum_{t=1}^{T} S_t^2 / s^2 \tag{11}$$

[4] In this case, under the null hypothesis of level-stationarity, the residuals of the regression of y_t on an intercept are the following: $e_t = y_t - \bar{y}$, $t = 1, 2, \ldots, T$.
[5] Saikkonen and Lukkonen (1990) derive a statistic of the same form as the locally best unbiased invariant test of the hypothesis $\theta = -1$ in the model $\Delta y_t = \nu_t + \theta \nu_{t-1}$ with $E(y_0)$ unknown and playing the role of intercept, with ν_t i.i.d. normal.
[6] The Bartlett window $w(s, l) = 1 - s/(l + 1)$ is used as in Newey and West (1987), which guarantees the nonnegativity of $s^2(l)$.
[7] The symbol $\hat{\eta}_\tau$ will be used if the regression includes a deterministic trend, and the symbol $\hat{\eta}_\mu$ if it only includes an intercept.
[8] In the KPSS test it is considered $p = 0$.
[9] For $\sigma_u^2 < \infty$, $0 < \theta < 1$.
[10] The symbol $\hat{\nu}_\tau$ will be used if the regression includes a deterministic trend, and the symbol $\hat{\nu}_\mu$ if it only includes an intercept.

where s^2 is a consistent estimator of the long term variance of ε_t, and S_t^2 is the partial sum of squared residuals e_t. Using this statistic, the null hypothesis of stationarity is rejected if it exceeds its critical value under the H_0.

KPSS and LMC tests differ in their treatment of the serial correlation under the H_0. Whereas the KPSS test uses a nonparametric correction similar to the Phillips–Perron test, the LMC test allows for autoregressive lags similar to the augmented Dickey–Fuller test, by Said and Dickey (1984). These differences imply that the KPSS test uses a nonparametric estimator of the long-term variance of ε_t. Although both tests have the same distribution function, the LMC test statistic converges at a rate $O_p(T)$ compared to $O_p(T/l)$ for the KPSS statistic. Furthermore, the LMC test is robust to the choice of the lag order, whereas the KPSS test is sensitive to the choice of l.[11]

III. STRUCTURAL CHANGES IN THE TIME SERIES COMPONENTS

Change in the intercept of a time series

In columns $\hat{\eta}_\mu$, from Tables 1 to 3, and in columns $\hat{\nu}_\mu$, from Tables 4 to 6, are shown the rejection rates (at the nominal significance level of 5%) of the H_0 of stationarity around an intercept. In columns $\hat{\eta}_\tau$, from Tables 1 to 3, and in columns $\hat{\nu}_\tau$, from Tables 4 to 6, is provided the rejection rates of the null of stationarity around a trend, when the true d.g.p. is stationary with a change in the intercept of size[12] α, at the moment δT, for sample sizes of 50, 100 and 300, respectively.[13]

Tables 1 to 3 show the results of the KPSS test. In general terms, the higher the sample size (T), the higher the spurious rejection rate of the level and trend stationarity H_0, irrespective of δ and α values. Specifically, looking at the results in columns $\hat{\eta}_\mu$, it is verified that the closer the break point to the middle of the time period, the higher the rejection rate. Furthermore, when the break point is close to the end or to the middle of the sample, then, the higher the α, the higher the spurious rejection of the level stationarity H_0. Opposite to that, in the rest of the sample the rate of spurious rejection decreases as α increases. The values in columns $\hat{\eta}_\tau$ show that the closer the break point to the middle or to the end of the sample, the lower the rejection rate of the trend stationarity hypothesis. Hence, the behaviour of the KPSS test with regard to the spurious rejection of the trend stationarity H_0, exhibits a wave-like shape, which is referred to as the *M effect*, as it is shown in Fig. 1. Additionally, if the break point is not around the

Table 1. *Rejection rate of H_0 with a structural change in the level (T = 50) (KPSS)*

δ	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$
	$\alpha = 2.5$		$\alpha = 5.0$		$\alpha = 10.0$	
0.05	0.0513	0.1437	0.0625	0.2355	0.0189	0.2652
0.10	0.0678	0.2259	0.0195	0.2514	0.0001	0.1787
0.20	0.0694	0.2042	0.0090	0.1740	0.0000	0.0567
0.30	0.5021	0.1131	0.1246	0.0389	0.0194	0.0008
0.40	0.4555	0.0105	0.6108	0.0001	0.7937	0.0000
0.50	0.5727	0.0010	0.8145	0.0000	0.9817	0.0000
0.60	0.4515	0.0072	0.6109	0.0000	0.7934	0.0000
0.70	0.1994	0.1137	0.1236	0.0438	0.0191	0.0011
0.80	0.0676	0.2010	0.0081	0.1700	0.0000	0.0599
0.90	0.0679	0.2219	0.0193	0.2509	0.0001	0.1772
0.99	0.0297	0.0803	0.0540	0.1531	0.0560	0.2375

Table 2. *Rejection rate of H_0 with a structural change in the level (T = 100) (KPSS)*

δ	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$
	$\alpha = 2.5$		$\alpha = 5.0$		$\alpha = 10.0$	
0.01	0.0381	0.0520	0.0588	0.0892	0.0842	0.1534
0.05	0.1083	0.1900	0.0884	0.2848	0.0203	0.3163
0.10	0.1642	0.3828	0.1112	0.5841	0.0203	0.7972
0.20	0.7025	0.8296	0.9442	0.9963	0.9996	1.0000
0.30	0.9942	0.7484	1.0000	0.9794	1.0000	1.0000
0.40	0.9998	0.1284	1.0000	0.0635	1.0000	0.0030
0.50	1.0000	0.0001	1.0000	0.0000	1.0000	0.0000
0.60	1.0000	0.1294	1.0000	0.0651	1.0000	0.0031
0.70	0.9950	0.7513	1.0000	0.9756	1.0000	1.0000
0.80	0.7060	0.8297	0.9436	0.9955	0.9995	1.0000
0.90	0.1699	0.3722	0.1116	0.5790	0.0197	0.7989
0.99	0.0366	0.0479	0.0577	0.0819	0.0825	0.1477

Table 3. *Rejection rate of H_0 with a structural change in the level (T = 300) (KPSS)*

δ	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$
	$\alpha = 2.5$		$\alpha = 5.0$		$\alpha = 10.0$	
0.01	0.0690	0.0766	0.1025	0.1380	0.1144	0.2272
0.05	0.2717	0.5141	0.3673	0.8564	0.4125	0.9966
0.10	0.9226	0.9970	0.9996	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	0.9992	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	0.9101	1.0000	1.0000	1.0000	1.0000
0.60	1.0000	0.9977	1.0000	1.0000	1.0000	1.0000
0.70	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.80	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.90	0.9203	0.9954	0.9997	1.0000	1.0000	1.0000
0.99	0.0695	0.0783	0.1004	0.1407	0.1153	0.2291

[11] See Leybourne and McCabe (1994) and Lee (1996).
[12] α values are arbitrary, but they are the same as in Leybourne *et al.* (1998).
[13] All the calculations have been programmed in Ox 2.0, London: Timberlake Consultants Ltd and Oxford: www.nuff.ox.ac.uk/Users/Doornik (Doornik, 1998), and simulations have been based on 10,000 iterations for each case under analysis.

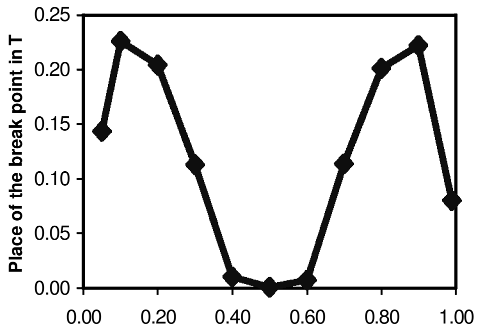

Fig. 1. *Rejection rate of the trend stationarity H_0 with a break point in the level. KPSS (α = 2.5 and T = 50)*

middle values of the sample, the higher the α, the higher the spurious rejection of the trend stationarity null, whereas the closer the break to the middle of the sample, the lower the spurious rejection rate.

Tables 4 to 6 show the behaviour of the LMC test. Broadly speaking, the higher the T, the higher the spurious rejection rate irrespective of δ and α values,[14] as in the KPSS test. When there is a change in the intercept of the series (columns $\hat{\nu}_\mu$), LMC test behaves as the KPSS test. The columns $\hat{\nu}_\mu$ show that, like in the KPSS test, the closer the break point to the middle of the sample period the higher the rejection rate of the level stationarity H_0, provided that a change in the intercept of the series has occurred. However, the spurious rejection rate becomes larger as α increases, irrespective of the location of the break point in the sample period. This result differs, therefore, from the KPSS test. In columns $\hat{\nu}_\tau$, we show that, for a given α, the spurious rejection rate exhibits an *M effect*, just in the same way as in the KPSS test (columns $\hat{\eta}_\tau$), although the rejection rates are higher than the KPSS test, as it can be seen in Fig. 2. Besides, in general terms, the spurious rejection phenomenon becomes more severe as α increases.

Finally, the simulation results, shown in Tables 1 to 6, reveal that the LMC test rejects the level (trend) stationarity hypothesis with higher likelihood than the KPSS one, when it is true. These differences become smaller as T increases.

Change in the trend of a time series

In this subsection, the effect of a structural change in the slope of the trend on the spurious rejection of the level or trend stationarity H_0 is analysed, when the KPSS and LMC tests are applied.

As in the previous subsection, the analysis is carried out for the sample sizes $T = 50$, $T = 100$ and $T = 300$, with the

[14] α values are those in Tables 1 to 3.

Table 4. *Rejection rate of H_0 with a structural change in the level (T = 50) (LMC)*

δ	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$
	$\alpha = 2.5$		$\alpha = 5.0$		$\alpha = 10.0$	
0.05	0.1365	0.2212	0.2909	0.4871	0.5544	0.7473
0.10	0.6449	0.7185	0.9406	0.9689	0.9525	0.9816
0.20	0.9858	0.9284	0.9987	0.9975	0.9956	0.9971
0.30	0.9992	0.9017	0.9999	0.9959	0.9920	0.9924
0.40	0.9998	0.7470	0.9999	0.9793	0.9998	0.9725
0.50	1.0000	0.5999	0.9999	0.9342	1.0000	0.9588
0.60	1.0000	0.7468	1.0000	0.9686	0.9999	0.9677
0.70	1.0000	0.9042	1.0000	0.9954	0.9999	0.9916
0.80	0.9914	0.9345	0.9997	0.9983	0.9989	0.9979
0.90	0.6423	0.7193	0.9714	0.9845	0.9912	0.9983
0.99	0.0754	0.0978	0.1040	0.1416	0.1358	0.2200

Table 5. *Rejection rate of H_0 with a structural change in the level (T = 100) (LMC)*

δ	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$
	$\alpha = 2.5$		$\alpha = 5.0$		$\alpha = 10.0$	
0.01	0.0471	0.0667	0.0398	0.0638	0.0468	0.1010
0.05	0.4114	0.5726	0.9127	0.9787	0.9868	0.9964
0.10	0.9705	0.9788	1.0000	1.0000	1.0000	0.9998
0.20	1.0000	0.9990	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	0.9992	1.0000	0.9999	1.0000	1.0000
0.40	1.0000	0.9838	1.0000	1.0000	1.0000	0.9999
0.50	1.0000	0.9413	1.0000	0.9999	1.0000	0.9998
0.60	1.0000	0.9816	1.0000	1.0000	1.0000	0.9999
0.70	1.0000	0.9984	1.0000	1.0000	1.0000	0.9998
0.80	1.0000	0.9994	1.0000	1.0000	1.0000	1.0000
0.90	0.9698	0.9736	0.9127	0.9787	1.0000	1.0000
0.99	0.0574	0.0766	0.0791	0.1051	0.1160	0.1741

Table 6. *Rejection rate of H_0 with a structural change in the level (T = 300) (LMC)*

δ	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$
	$\alpha = 2.5$		$\alpha = 5.0$		$\alpha = 10.0$	
0.01	0.0747	0.0937	0.1304	0.2113	0.5045	0.7736
0.05	0.9672	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.70	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.80	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.90	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.99	0.0819	0.1000	0.1456	0.2330	0.4196	0.7317

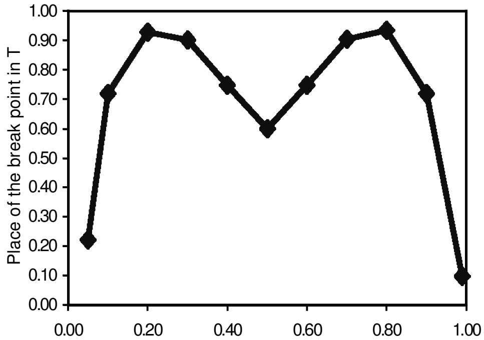

Fig. 2. *Rejection rate of the trend stationarity H_0 with a break point in the level. LMC ($\alpha = 25$ and $T = 50$)*

Table 7. *Rejection rate of H_0 with a structural change in the trend ($T = 50$) (KPSS)*

δ	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$
	$\beta = 0.5$		$\beta = 1.0$		$\beta = 2.0$	
0.05	1.0000	0.0605	1.0000	0.0992	1.0000	0.1813
0.10	1.0000	0.1900	1.0000	0.2787	1.0000	0.2897
0.20	1.0000	0.3662	1.0000	0.4189	1.0000	0.4340
0.30	1.0000	0.7596	1.0000	0.9587	1.0000	1.0000
0.40	1.0000	0.9900	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	0.9992	1.0000	1.0000	1.0000	1.0000
0.60	1.0000	0.9787	1.0000	1.0000	1.0000	1.0000
0.70	1.0000	0.6719	1.0000	0.8879	1.0000	0.9935
0.80	1.0000	0.3371	1.0000	0.3726	1.0000	0.3488
0.90	1.0000	0.1396	1.0000	0.2400	1.0000	0.2914
0.99	1.0000	0.0473	1.0000	0.0473	1.0000	0.0473

following changes in the slope:[15] $\beta = 0.5$, $\beta = 1.0$ and $\beta = 2.0$. The simulation results[16] using the KPSS test are reported in Tables 7 to 9 and those using the LMC test are reported in Tables 10 to 12. Columns $\hat{\eta}_\mu$, in Tables 7 to 9, and $\hat{\nu}_\mu$, in Tables 10 to 12, show the spurious rejection rates (at the nominal significance level of 5%) of the H_0 of stationarity around a level. Columns $\hat{\eta}_\tau$, in Tables 7 to 9, and $\hat{\nu}_\tau$, in Tables 10 to 12, report the rejection rates of the trend stationarity H_0, when the true d.g.p. is stationary.

In Tables 7 to 9, it is shown that, if there is a change in the slope, the level stationarity H_0 is always rejected by the KPSS test, irrespective of the sample size, the location and the magnitude of the break point. On the other hand, if the trend stationarity H_0 is tested, the spurious rejection rate of H_0 becomes larger whether both the sample size or β increases and/or when the break point is close to the middle of the sample.

Results in columns $\hat{\nu}_\mu$, reported in Tables 10 to 12, show that when the LMC test is applied, the rejection rate of the level stationarity H_0 increases as both the change in the slope and T become larger, provided that the break point is not located at the end of the sample. When it is located at the end of the sample, the rejection rate decreases as β increases. Additionally, testing the trend stationarity H_0, when it is true, leads to a rejection rate which is increasing with sample size. Besides, if both the sample and the break size are kept constant, the spurious rejection rate becomes larger as the break point is closer to the middle of the sample. The β parameter is also an important criterion to test the trend stationarity hypothesis when using the LMC test, since as β increases, the spurious rejection rate decreases, unless the break point is close to the end of the sample.

Roughly speaking, results in Tables 7 to 12 show that, when there is a structural change in the slope, the KPSS

Table 8. *Rejection rate of H_0 with a structural change in the trend ($T = 100$) (KPSS)*

δ	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$
	$\beta = 0.5$		$\beta = 1.0$		$\beta = 2.0$	
0.01	1.0000	0.0418	1.0000	0.0427	1.0000	0.0496
0.05	1.0000	0.1246	1.0000	0.2194	1.0000	0.2891
0.10	1.0000	0.3949	1.0000	0.5735	1.0000	0.7739
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.70	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.80	1.0000	0.9991	1.0000	1.0000	1.0000	1.0000
0.90	1.0000	0.3380	1.0000	0.4844	1.0000	0.6244
0.99	1.0000	0.0413	1.0000	0.0413	1.0000	0.0413

Table 9. *Rejection rate of H_0 with a structural change in the trend ($T = 300$) (KPSS)*

δ	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$
	$\beta = 0.5$		$\beta = 1.0$		$\beta = 2.0$	
0.01	1.0000	0.0477	1.0000	0.0660	1.0000	0.1147
0.05	1.0000	0.6836	1.0000	0.9393	1.0000	0.9996
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.70	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.80	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.90	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.99	1.0000	0.0440	1.0000	0.0474	1.0000	0.0645

15 β values in the simulation are arbitrary, although identical to those used in Leybourne *et al.* (1998).

16 All the calculations have been programmed in Ox 2.0, London: Timberlake Consultants Ltd and Oxford: www.nuff.ox.ac.uk/Users/Doornik (Doornik, 1998), and simulations have been based on 10 000 iterations for each case under analysis.

Table 10. *Rejection rate of H_0 with a structural change in the trend (T = 50) (LMC)*

δ	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$
	$\beta = 0.5$		$\beta = 1.0$		$\beta = 2.0$	
0.05	0.1310	0.0773	0.0843	0.1144	0.0370	0.3356
0.10	0.1012	0.3582	0.0843	0.1144	0.6317	0.9448
0.20	0.0772	0.9959	0.2414	0.9949	1.0000	0.7386
0.30	0.0736	0.9996	0.7161	0.9885	0.9999	0.5304
0.40	0.0920	0.9996	0.9023	0.9843	0.9999	0.4247
0.50	0.1109	0.9993	0.9239	0.9831	1.0000	0.4034
0.60	0.1228	0.9996	0.8445	0.9867	1.0000	0.4679
0.70	0.1520	0.9996	0.5281	0.9909	0.9998	0.6351
0.80	0.1941	0.9919	0.1861	0.9977	0.9913	0.8702
0.90	0.2616	0.1696	0.1981	0.5444	0.1979	0.9857
0.99	0.3147	0.0824	0.3147	0.0824	0.3147	0.0824

Table 11. *Rejection rate of H_0 with a structural change in the trend (T = 100) (LMC)*

δ	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$
	$\beta = 0.5$		$\beta = 1.0$		$\beta = 2.0$	
0.01	0.1826	0.0623	0.2234	0.0558	0.1996	0.0467
0.05	0.1335	0.1948	0.0857	0.7562	0.0270	0.9990
0.10	0.0909	0.9965	0.0126	1.0000	0.9914	0.9930
0.20	0.1401	0.3756	0.8735	0.9991	0.9985	0.8849
0.30	0.0306	0.9999	0.9994	0.9975	0.9993	0.6418
0.40	0.0784	1.0000	0.9997	0.9969	0.9988	0.4660
0.50	0.1110	1.0000	0.9995	0.9970	0.9995	0.4216
0.60	0.0766	0.9999	0.9995	0.9971	0.9993	0.5016
0.70	0.0638	1.0000	0.9991	0.9978	0.9997	0.7080
0.80	0.0896	1.0000	0.6075	0.9997	0.9960	0.9372
0.90	0.1441	0.9821	0.0967	1.0000	0.7819	0.9981
0.99	0.2208	0.0681	0.2208	0.0681	0.2208	0.0681

Table 12. *Rejection rate of H_0 with a structural change in the trend (T = 300) (LMC)*

δ	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$	$\hat{\nu}_\mu$	$\hat{\nu}_\tau$
	$\beta = 0.5$		$\beta = 1.0$		$\beta = 2.0$	
0.01	0.3318	0.0500	0.4486	0.0513	0.4757	0.0968
0.05	0.2333	1.0000	0.1349	1.0000	0.9512	1.0000
0.10	0.1218	1.0000	0.8279	1.0000	0.9585	1.0000
0.20	0.1193	1.0000	0.9932	1.0000	0.9958	0.9934
0.30	0.9895	1.0000	0.9941	1.0000	0.9957	0.8331
0.40	0.9917	1.0000	0.9932	1.0000	0.9919	0.5599
0.50	0.9940	1.0000	0.9948	1.0000	0.9785	0.4638
0.60	0.9943	1.0000	0.9960	1.0000	0.9871	0.5804
0.70	0.9858	1.0000	0.9966	1.0000	0.9958	0.8590
0.80	0.0342	1.0000	0.9962	1.0000	0.9958	0.9663
0.90	0.0221	1.0000	0.3837	1.0000	0.9985	1.0000
0.99	0.1134	0.0527	0.0948	0.0529	0.0499	0.0665

test rejects the null 'stationarity around a level' more often than the LMC test. However, the LMC test rejects the null 'stationarity around a slope' more frequently than the KPSS test, although the differences between both tests become smaller as the sample size increases.

As a conclusion, if there is a structural change in the intercept or in the slope, the KPSS and LMC rejection rates are larger than the nominal size.

IV. SUMMARY

Time series do not provide enough information to detect unit roots. As a matter of fact, unit root tests are not very powerful against near unit root processes, so it may be useful to test the stationarity hypothesis as in the KPSS and LMC tests. However, in this work it has been shown that those procedures may exhibit seriously distorted size, when there is a change in the level or in the trend of the DGP. This result depends on the sample size, the location of the break point in the sample and the size of the break.

REFERENCES

Banerjee, A., Lumsdaine, R. L. and Stock, J. H. (1992) Recursive and sequential tests of the unit root and trend break hypothesis: theory and international evidence, *Journal of Business and Economic Statistics*, **10**, 271–87.

Campos, J., Ericsson, N. R. and Hendry, D. F. (1996) Cointegration tests in the presence of structural breaks, *Journal of Econometrics*, **70**, 187–220.

Christiano, L. J. (1988) Searching for a break in GNP, *working paper 416*, Federal Reserve Bank of Minneapolis.

Dickey, D. A. and Fuller, W. A. (1979) Distribution of the estimators for autoregressive time series with a unit root, *Journal of the American Statistical Association*, **74**, 427–31.

Doornik, J. A. (1998) Object-Oriented Matrix programming using Ox 2.0, Timberlake Consultants Ltd, London and Oxford: www.nuff.ox.ac.uk/Users/Doornik.

Harvey, A. C. (1989) *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press, Cambridge.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P. and Shin, A. (1992) Testing the null hypothesis of stationarity against the alternative of a unit root, *Journal of Econometrics*, **54**, 159–78.

Lee, J. (1996) On the power of stationarity tests using optimal bandwidth estimates, *Economics Letters*, **51**, 131–7.

Leybourne, S. J. and McCabe, B. P. M. (1994) A consistent test for a unit root, *Journal of Business and Economic Statistics*, **12**(2), 157–68.

Leybourne, S. J., Mills, T. C. and Newbold, P. (1998) Spurious rejections by Dickey–Fuller tests in the presence of a break under the null, *Journal of Econometrics*, **87**, 191–203.

Newey, W. K. and West, K. D. (1987) A simple, positive semi-definite, heteroscedasticity and autocorrelation consistent covariance matrix, *Econometrica*, **55**, 703–8.

Nyblom, J. (1986) Testing for deterministic linear trend in time series, *Journal of the American Statistical Association*, **81**, 545–9.

Nyblom, J. and Makelainen, T. (1983) Comparisons of tests for the presence of random walk coefficients in a simple linear model, *Journal of the American Statistical Association*, **78**, 856–64.

Perron, P. (1989) The Great Crash, the oil price shock and the unit root hypothesis, *Econometrica*, **57**, 1361–401.

Perron, P. (1993) The Great Crash, the oil price shock and the unit root hypothesis: Erratum, *Econometrica*, **61**, 248–9.

Perron, P. (1994) Trend, unit root and structural change in macroeconomic time series, in *Cointegration for the Applied Economist* (Ed.) B. B. Rao, Macmillan, New York, NY, 113–46.

Phillips, P. C. B. (1987) Time series regression with unit roots, *Econometrica*, **55**, 277–301.

Phillips, P. C. B. and Perron, P. (1988) Testing for a unit root in time series regression, *Biometrika*, **75**, 335–46.

Said, S. E. and Dickey, D. A. (1984) Testing for unit roots in autoregressive-moving average models of unknown order, *Biometrika*, **71**, 599–608.

Saikkonen, P. and Lukkonen, R. (1990) Testing for a moving average unit root, *working paper*, Department of Mathematics, University of Helsinki.